

# Numerical analysis on heat removal from *Y*-shaped fins: Efficiency and volume occupied for a new approach to performance optimisation

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## Abstract

This paper faces the problem of geometric optimisation for exchanger profiles with innovative shapes. In particular it was analysed the *Y* shape, keeping the dimensionless thermal conductance as reference parameter like in the thematic technical literature. The use of suitable geometrical constraints allows wide comparisons with papers in literature using different kinds of geometries.

The methodological approach here chosen is numerical and utilises a CFD software. The geometries examined are obtained by varying the angle between the two arms of the *Y*, starting from the *T*-shaped profile that allows the best performances, as obtained in a previous work. Results show that the new shape proposed for the fins, together with the assessment of the horizontal width, leads to a novel performance evaluation criterion.

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**Keywords:** *Y*-shaped fins; Constructal theory; Optimisation; Efficiency; CFD

## 1. Introduction

The recent evolutions in electronic and industrial components designing, enhanced the role played by heat exchangers significantly. An important example is given by information technology, where an evident trend, predicted by Gordon E. Moore's law [1], shows that the power of computer micro-processors doubles every eighteen months, keeping constant their volume. The necessity of ever higher specific powers leads to increasing the number of the modular computing units inserted in the surface allocated to the CPU, which has an order of magnitude of a square centimetre. A so intense increase of performances determines a consequent enhancement in the thermal power generated by the components. This leads to the need of an ever more effective cooling, to avoid malfunctions or degradations of the operating systems [2]. It follows that optimised shapes are to be searched [3,4] to allow better heat transfer performances.

The many possible experimental approaches require to materially realise the models to test. This causes a significant use of resources, both economic and of time, as even a single modification to a test parameter makes it necessary a new physic realisation of the model. It is so clear that those studies are affected by the researcher's possibilities and the results obtained are conditioned by the experimental error, which is never completely avoidable [5–7].

For these reasons many researchers have recently preferred numerical approaches to the problem. Among them, in [8] it is used the CFX code to study forced convection in turbulent conditions inside a cylindrical duct with longitudinal fins. In [9], instead, it is studied the optimisation of heat exchangers for electronic componentry use: a finite volume approach allows for the reduction of the otherwise relevant empirical contribution that characterises these processes and that often damages the economic results of the exchange process. In [10] CFD techniques are applied in defining the performance of auxiliary heat removal devices applied to electronic packaging: the approach proves its effectiveness. This paper instead can be regarded as an evolution of previous studies on performance optimisation of *T*-shaped fin-based heat exchangers. Those studies relate to

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## Nomenclature

$a$	dimensionless parameter, Eq. (3)
$A$	area ..... $\text{m}^2$
$h$	heat transfer coefficient ..... $\text{W m}^{-2} \text{K}^{-1}$
$k_1, k_2$	dimensionless ratios
$L$	length ..... $\text{m}$
$q$	heat ..... $\text{W}$
$t$	thickness ..... $\text{m}$
$T$	temperature ..... $\text{K}$
$V$	volume ..... $\text{m}^3$
$W$	width ..... $\text{m}$

## Greek letters

$\alpha$	angle between the arms of the Y-shaped fin .... $^\circ$
$\phi$	volume ratio of fin material, Eq. (2)
$\lambda$	fin thermal conductivity ..... $\text{W m}^{-1} \text{K}^{-1}$

## Subscripts

$f$	related to the fin
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## Superscript

(*)	dimensionless variables, Eq. (1)
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Bejan's constructal theory, powerful method useful to describe many systems and conditions in different fields. The first reference taken into account was that quoted as [11], where the authors seek an optimised configuration of  $T$ - and  $Y$ -shaped channels for fluid flows, in function of some fundamental ratios related to their geometries. The main reference, used as source of this work is, anyway, that in [12], where it is applied the constructal theory to optimise the performance of a  $T$ -shaped fin, using as evaluation parameter the thermal conductance. The assessment is done considering some geometrical constraints characteristic of the problem. The numerical method used in this work is based on that defined in [13] and, starting from the geometry realised in a CAD environment, allows to study the effect of each influencing parameter, both geometrical and thermo-fluid mechanical. It uses a limited amount of resources and, not being affected by any systematic errors, gives wide technical applicability to the study.

In particular the research begins from the necessity to investigate new optimum configurations that allow further performance enhancements, starting from an already optimised system, in agreement with [12,13].

Aiming at this, it is here evaluated, being forced convection the heat transfer mechanism analysed, the effect of the angle  $\alpha$  between the two arms of the  $T$ , varying the profile of the fin towards a  $Y$  shape. That gives a noticeable added value to the method adopted, as an analogous investigation performed with experimental media would have required a huge amount of different systems to be tested, with a considerable amount of time consequently wasted.

## 2. Model definition

As indicated in the previous section, this work makes one step ahead with respect to that in [12]. From this quoted reference it is taken both the nomenclature and the characterisation of the optimal  $T$ -shaped fin. The latter was the starting point for new developments. Fig. 1 represents the computational domain, as in [12], with the related symbols.

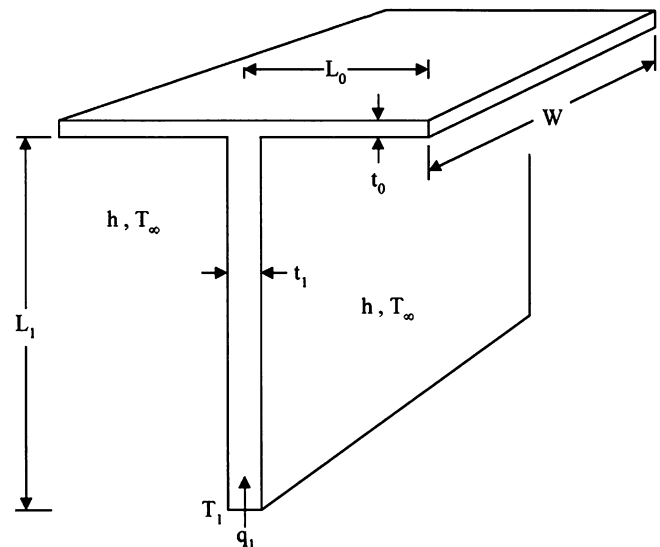


Fig. 1. Starting profile.

Moreover, the fin performance optimisation had, as parameter of evaluation, the dimensionless conductance  $q_1^*$  through the root of the  $T$ , defined as:

$$q_1^* = \frac{q_1}{\lambda W (T_1 - T_\infty)} \quad (1)$$

where  $q_1$  represents the thermal power through the root,  $\lambda$  is the thermal conductivity,  $W$  the third dimension representing the depth of the fin,  $T_1$  the root temperature and  $T_\infty$  the fluid temperature.

The optimal geometry, in dependence of the dimensionless ratios  $k_1 = t_1/t_0$  and  $k_2 = L_1/L_0$ , was that characterised by  $k_1 = 5$  and  $k_2 = 0.07$  [12].

Based on the nature of the geometrical variations obtained when varying the angle examined, three cases study can be put in evidence, even if only for technical and not theoretical convenience.

*Case I:*  $180^\circ < \alpha < \alpha_{\text{lim}}$ .

In this range of values for  $\alpha$ , the fin assumes the shape of an arrow (Fig. 2(a)). The angle  $\alpha_{\text{lim}}$  is the one that allows the

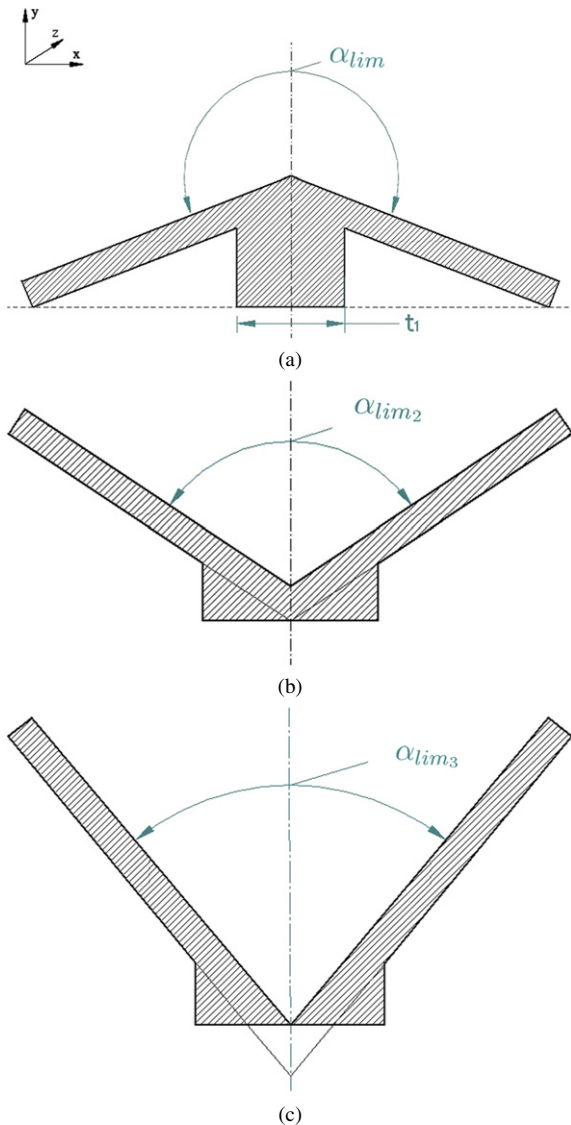


Fig. 2. Geometry cases I (a), II (b), III (c).

contact of the two arms of the profile with the support of the system. Obviously  $\alpha_{lim}$  is the highest value that can be assigned to  $\alpha$ ; for bigger values the arms of the fin would intersect the fin root plane.

**Case II:**  $\alpha_{lim2} < \alpha < 180^\circ$ .

In this range of values for  $\alpha$  (together with case III) the fin is given a Y shape (Fig. 2(b)). The smallest angle in this case is  $\alpha_{lim2}$ , value at which the intersection between the bottom surfaces of the arms is tangent to the support plane of the system. The highest value of  $\alpha$  in this range is typical of a T configuration.

**Case III:**  $\alpha_{lim3} < \alpha < \alpha_{lim2}$ .

In this range of values a further reduction of  $\alpha$  with respect to case II causes a virtual intersection between the two bottom surfaces of the arms under the fin root plane (Fig. 2(c)).

To be able to compare the results here obtained to those of the previously optimised T [11,12], it is necessary to respect the following geometrical constraint:

$$\varphi = \frac{A_f}{A} \quad (2)$$

where  $A_f$  and  $A$  are the cross sectional surface of the fin and the surface subtended by the two arms of the fin, respectively (Figs. 3(a) and 3(b)).

The expressions for  $A_f$  (Fig. 3(a)) and  $A$  (Fig. 3(b)) are obtained through trigonometric computations. They slightly differ from case to case because of the contribution of the extensions of the arms, variable as a consequence of the constraint chosen. As shown in Fig. 2(c), part of the parallelepipeds forming the arms of the profile “disappears” under the root of the system. This does not hold true for values of  $\alpha$  greater than  $\alpha_{lim2}$  and therefore it has a fundamental importance to consider separate cases in the geometrical definition of the fins.

Once matched this condition, it can be observed a trend of the arms length  $L_0$  in dependence of the value assumed by the angle  $\alpha$ . This relation is represented in Fig. 4, where the grey profile corresponds to the angle that identify an inversion in the trend of the length of the arms with  $\alpha$ , for a value of  $\alpha = 95^\circ$ .

The boundary conditions are defined by evaluating a convection coefficient  $h$  variable with respect to the dimensions of the fin as inversely proportional to  $A^{1/2}$ :

$$h = \frac{a^2 \lambda}{2\sqrt{A}} \quad (3)$$

The other computational parameters are the conduction coefficient  $\lambda$  and the dimensionless number  $a$ , used as arbitrary reference for the experimental variation of the convection coefficient. Its value is assumed constant and equal to 0.1 [11,12]. This assumption is valid in the actual case of a forced convection process with the refrigerant fluid flowing in the  $z$ -direction (see Fig. 2) at a very high speed.

It is then necessary to give the punctual values used for the definition of the parameters that the software Femlab requires as input data. The temperature  $T_1$  at the root of the system and that of the fluid  $T_\infty$  are set equal to 373.15 K and 293.15 K, respectively, because the software needs dimensional values. This assumption does not affect the generality of the study as the conductance, target value, is defined by means of a difference of temperature and is expressed, in dimensionless notation, as shown in (1).

For the same reason it is not reductive to consider, in accordance with [11], the thermal conductivity  $\lambda$  equal to  $200 \text{ W m}^{-1} \text{ K}^{-1}$ , value that, moreover, is typical of aluminium, often used in applications of thermo-fluiddynamic nature.

Once demonstrated its effect on the computational resources use, the third dimension  $W$  of the profile was set, after suitable sample testing to avoid any geometrical dependences, equal to the lowest value able to make negligible the side effects, that is to give to the system a hemi-infinite depth [11,12]. This choice was also fundamental to make the solver converge.

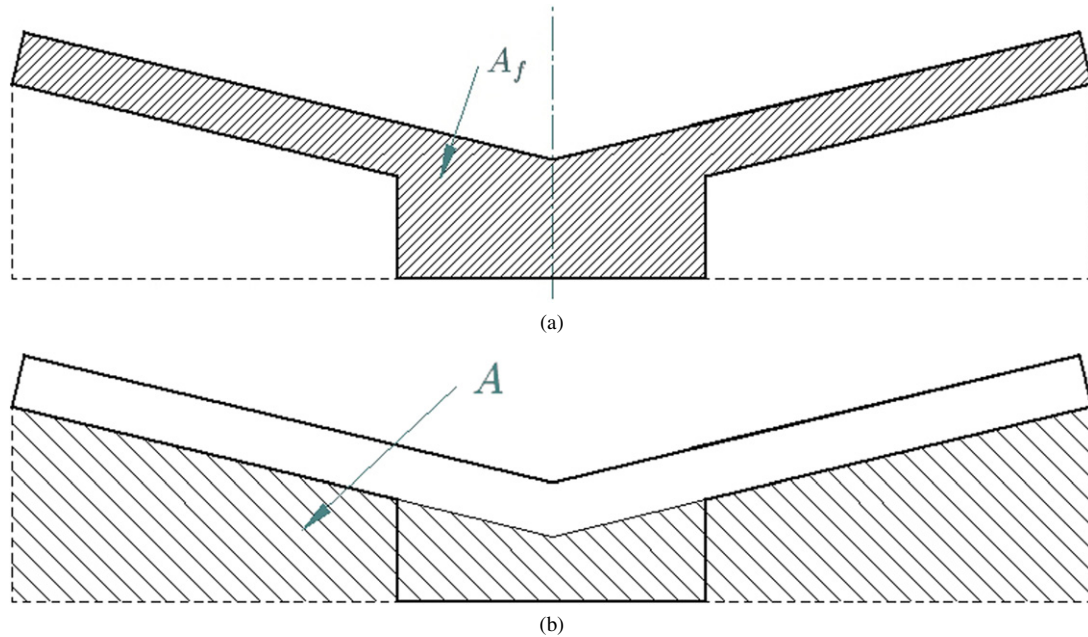


Fig. 3. Cross section  $A_f$  (a), Surface subtended  $A$  (b).

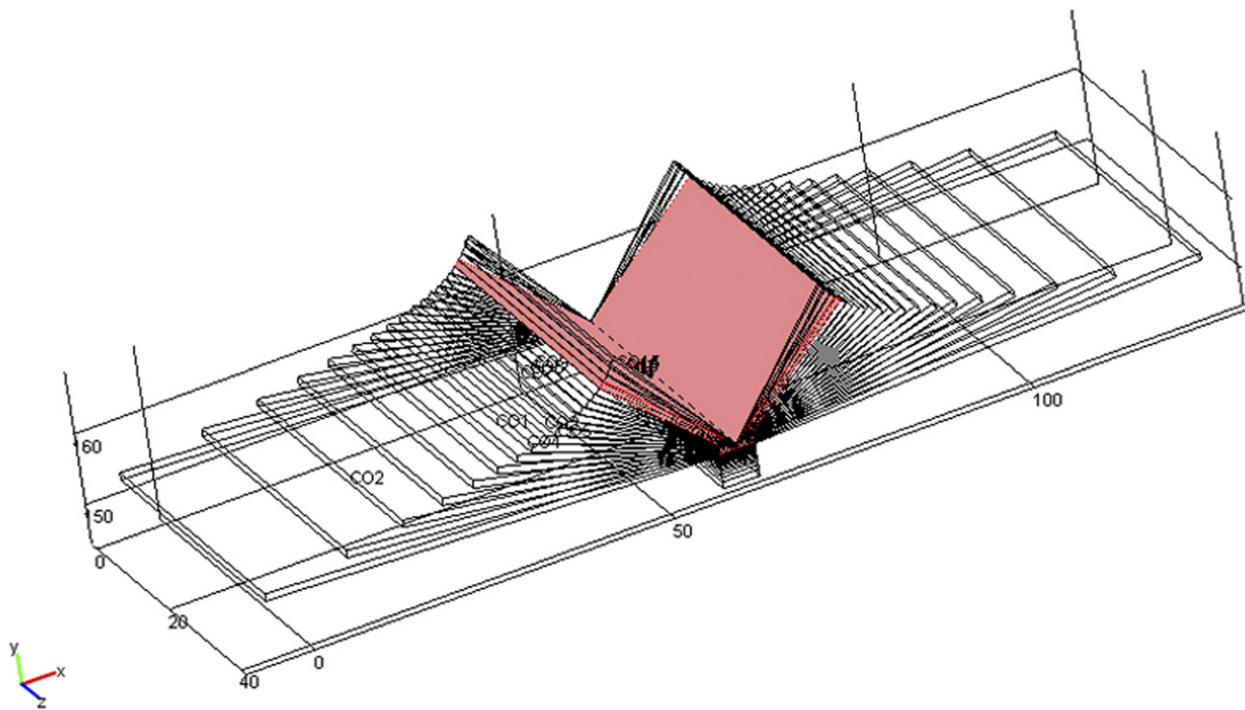


Fig. 4. Geometry in dependence of  $\alpha$  (dimensionless geometries;  $t_0 = 1$ , see Fig. 1).

The software employed in the numerical elaboration was Femlab 3.1, based on a simple graphical interface and able to generate with its own resources simple geometries. The geometries of the case studies here presented were realised thanks to an external CAD module, taking advantage of the properties of the IGES file format to share the files between the two softwares. Once imported the geometry in Femlab, it was built an opportune mesh (Fig. 5), with a variable number of ele-

ments, proportional case by case to the angle  $\alpha$ : its range varies between 34439 elements, for  $\alpha$  equal to  $100^\circ$ , and 223567 elements, for  $\alpha$  equal to  $188^\circ$ .

The mesh was, in each case, optimised to guarantee better refinement in highly critical zones. Among them the intersections between horizontal and vertical components of the whole geometry, that have shown high thermal gradients and that resulted critical also studying the  $T$  profiles [12].

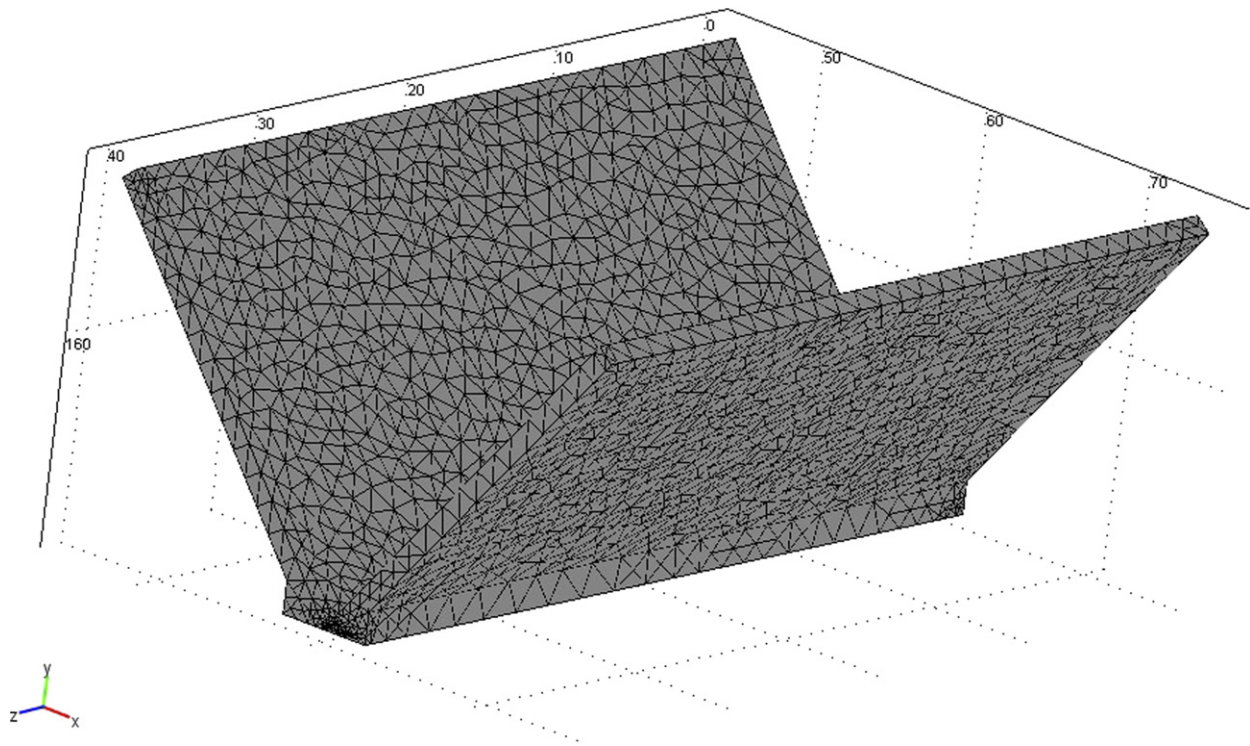


Fig. 5.  $Y$  domain: Example of mesh (dimensionless geometries;  $t_0 = 1$ , see Fig. 1).

### 3. Results and comments

As anticipated, the purpose of the present work is to assess, starting from the optimised profile obtained in [11,12], how to make possible a further geometrical optimisation of the system investigated in function of the angle  $\alpha$  between the two arms of the fin. In accordance to the reference studies, the main parameter on which to base the optimisation evaluation is the dimensionless conductance.

In relation to the CAD model created and to the boundary conditions chosen, the output of the software was a thermal field with a trend similar to the one in Fig. 6. The temperature distribution makes sense if compared to the  $T$  case in [11,12]: the highest temperature is in correspondence to the root and it gradually decreases.

Representing in a Cartesian diagram the trend of the dimensionless conductance in function of the angle  $\alpha$ , it can be obtained the graph reported in Fig. 7.

Such curve shows a variable trend highlighting a maximum and a minimum value. The range of values of  $q_1^*$  lies between a bit more than 0.025 and a bit less than 0.035. It is fundamental to observe that the target parameter is, as imagined, a function of  $\alpha$  and therefore the investigation makes fully sense. How it can be seen, the highest value of conductance is in correspondence of an angle  $\alpha$  equal approximately to  $180^\circ$  ( $T$ -shape). At first sight such result seemed not a positive one in the sense that no other one value for  $\alpha$  than the one typical of a  $T$ -shaped fin would enable enhancements of the thermal performance of the profile. If the assessment stopped here the conclusion would be that this research has characterised an important influential

parameter which, anyway, does not suggest any new geometry able to create a performance enhancement. Such evaluation, instead, would be incomplete because in the problems of technical interest a thermo-fluiddynamic optimisation of an heat exchange process cannot ever be separated from the “space factor”. In fact performances have always to be weighted in relation to the space available, as industry looks for high performance in limited spaces occupied by the device.

This is why the main result of this investigation comes just after a close examination of the efficiency of the fin. Efficiency is defined as the ratio of the heat exchanged in the actual case and the one exchangeable if all the surface of the fin were at the temperature of the root. Aiming at this analysis, it can be obtained the graph of Fig. 8, where the efficiency is reported as a function of  $\alpha$ .

The efficiency of the tested systems proves to decrease with increasing  $\alpha$ . In correspondence to values of  $\alpha$  smaller than  $100^\circ$  it reaches the vicinity of 1. This represents a result of great interest, as it integrates the performance assessment of the profiles investigated, till now in literature mainly focused on the dimensionless conductance. Efficiency then suggests a different and wider optimisation criterion that will have to be further deepened and applied in future researches. In addition to efficiency, the performance of a fin can also be assessed taking into account the whole surface by means of a parameter called effectiveness. Effectiveness is defined as the ratio of the fin heat transfer rate to the heat transfer rate that would exist without the fin [14]. Applying such a computation to the present case gives the results shown in Fig. 9, where effectiveness is plotted as a function of  $\alpha$ . The trend is somehow opposite that in



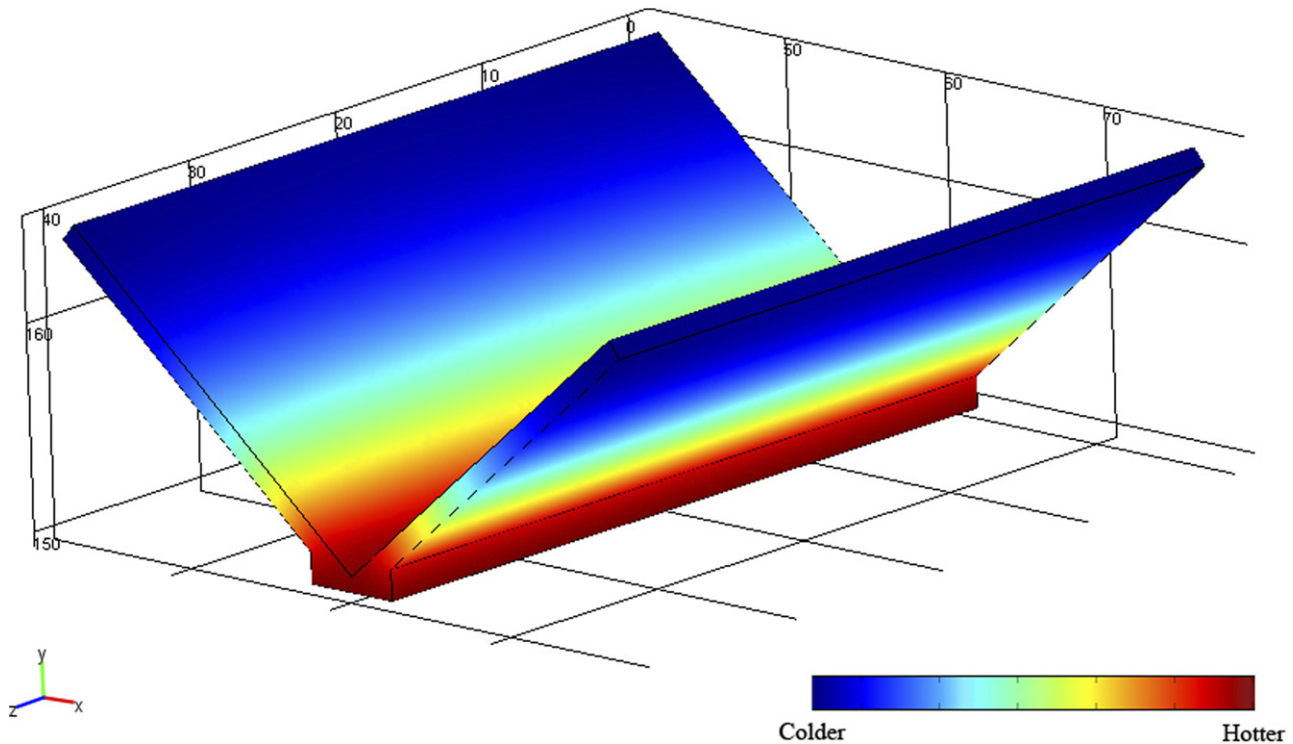
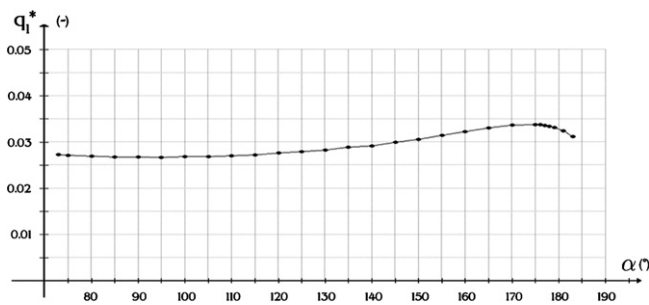
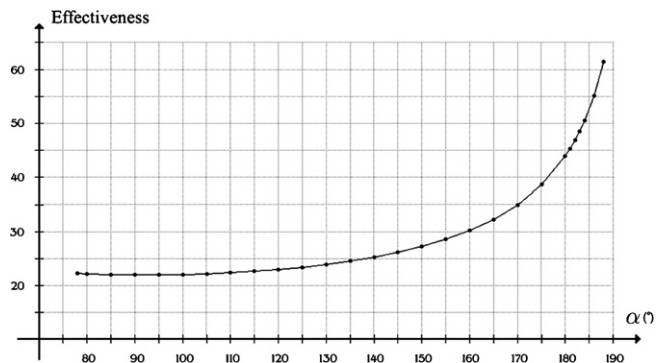
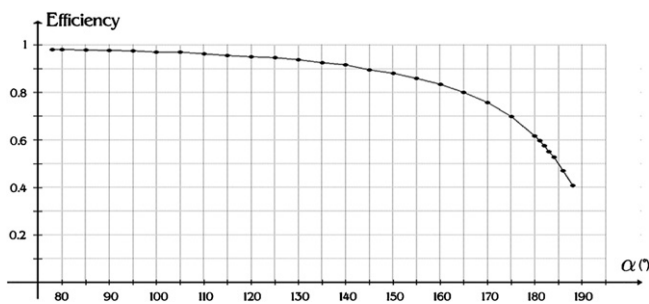
Fig. 6. Temperature trend in the fin (dimensionless geometries;  $t_0 = 1$ , see Fig. 1).Fig. 7. Trend of  $q_l^*$  with  $\alpha$ .Fig. 9. Effectiveness of the fins in function of  $\alpha$ .Fig. 8. Efficiency of the fins in function of  $\alpha$ .

Fig. 8 (efficiency) but this could be supposed simply looking at the geometries investigated. What is essential, anyway, is that effectiveness proves to be always greater than 20 which, being 2 the limit value to evaluate the convenience of a fin with respect to a flat surface, shows how “thermally convenient” the process analysed is for any of the geometries considered.

#### 4. Conclusions

In every energetic system a part of the energy utilised is converted in undesired heat. Electronic devices and heat exchangers industries are particularly sensitive to this issue as they live a natural trend towards ever more compact and efficient systems. Such an aim necessarily generates many realisation difficulties, function of the integration level between the single subsystems, that increases as a consequence of the more complicate and performing systems required.

In this field of application the present work has proposed a numerical approach to the problem, using a finite element CFD code. The aim of the research was to identify a possible geometrical evolution of the T-shaped finned profiles optimised, in a previous work taken as main reference, with respect to the value of the thermal conductance. Starting from the reference

optimal results, it was studied the trend of the assessment parameter (the dimensionless conductance  $q_1^*$ ) in function of the angle  $\alpha$  between the two horizontal arms of the fin.

If, on the one hand, it has not been found a value for  $\alpha$ , different from  $180^\circ$ , capable of a higher value of  $q_1^*$ , on the other a close exam of the thermal efficiency showed a significant increase of this parameter. This holds especially true in correspondence to those values of  $\alpha$  smaller than  $100^\circ$ . This evidence has concurred to a new definition of optimisation, gained by an introduction of the fundamental "space factor". The present work so unified the "classic" definitions of optimisation and efficiency in a new general performance criterion.

In fact, the trend determined for the efficiency in function of  $\alpha$  is associated to a relevant reduction of the horizontal width of the  $Y$ , decreasing with increasing  $\alpha$ . Such result opens a new perspective, at the moment in progress, on multiple-fin systems. Concluding, future numerical constructal design work on  $Y$ -shaped fins will be necessary, accounting for convection in the fluid space, for all angles. Therefore it will be of great importance to evaluate those cases with variable heat transfer coefficient, natural or forced convection and single- or multi-fin systems, evaluating different flow conditions and cooling fluids. The authors are at the moment working on all these issues.

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